TEMPERATURE DISTRIBUTION IN A SYSTEM OF TWO CIRCULAR PLATES

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The article presents an approximate analytical solution of the one-dimensional axisymmetrical steady-state problem for the radial temperature distribution in the case of a system consisting of two circular plates of equal radius and different thicknesses. It is assumed that the plates are attached to supports through spacer-mounts and are subject to the conditions of heat transfer by thermal conduction, convection, and radiation both with one another and with a surrounding medium of known temperature. The solution is obtained for boundary conditions of the first and third kinds.

The problem reduces to the solution of the set of nonlinear differential equations

$$\frac{d^2T_1}{dr^2}+\frac{1}{r}\cdot\frac{dT_1}{dr}+\frac{1}{\lambda_1 h_1}(-a_1T_1^4+b_1T_2^4-c_1T_1+d_1T_2+e_1)=0\ ,$$
$$\frac{d^2T_2}{dr^2}+\frac{1}{r}\cdot\frac{dT_2}{dr}+\frac{1}{\lambda_2 h_2}(-a_2T_2^4+b_2T_1^4-c_2T_2+d_2T_1+e_2)=0\ . \quad (1)$$

The boundary conditions of the first kind (known temperatures U_1 and U_2 at the edges of the plates) and third kind (known temperatures V_1 and V_2 of the plate supports) have the respective forms

$$T_1(r)\,|_{r=R}=U_1,\quad T_2(r)\,|_{r=R}=U_2\,; \quad (2)$$

$$\lambda_1\left.\frac{dT_1}{dr}\right|_{r=R}=\mu_1^{-1}(V_1-T_1),\quad \lambda_2\left.\frac{dT_2}{dr}\right|_{r=R}=\mu_2^{-1}(V_2-T_2)\,. \quad (3)$$

According to the symmetry condition $dT_1/dr = dT_2/dr = 0$ at $r = 0$.

After linearization $T_1(r) = \theta_1 + t_1(r)$, $T_2(r) = \theta_2 + t_2(r)$ [where θ_1 and θ_2 are constants to be determined and $t_1(r)$ and $t_2(r)$ are the new variables] and transformation to dimensionless coordinates $\rho = r/R$ the solution of the system (1) is obtained in the form

$$T_1(\rho)=\theta_1+Mm_2I_0(k\rho)+Nn_2I_0(l\rho)\,,$$
$$T_2(\rho)=\theta_2+m_2I_0(k\rho)+n_2I_0(l\rho)\,. \quad (4)$$

The constants θ_1 and θ_2 are defined as the steady-state temperatures for total thermal insulation of the lateral surfaces of the plates. Expressions for the constants M, m_2, N, n_2, k, and l are given in the article as a function of the boundary conditions (2) and (3). The acceptable engineering accuracy of the approximate analytical solution (4) is demonstrated by comparison with results obtained by the method of finite differences on a BÉSM-4 digital computer.

NOTATION

r	is the radial coordinate on the plates;
R	is the total radius of the plates;
T(r)	is the temperature;
h	is the plate thickness;
λ	is the thermal conductivity;
a, b, c, d, e	are the constants depending on the design data and heat-transfer conditions on the plate surfaces;

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μ is the thermal resistance of the mounts;

I_0 is the Bessel function of an imaginary argument.

Subscripts

1, 2 are the order numbers of the plates.